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A COMPUTER PROGRAM TO DETERMINE THE CRITICAL SPEED OF ROTATING MACHINERY

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ABSTRACT

There is a computer program available for use on Cyber which allows one to determine the critical speeds, normal mode shapes and forced vibration response of any piece of rotating machinery.[1] The program is based on the Holzer-Mykelstad-Prohl Method modified to apply to lateral vibrating systems. This program cannot be applied to torsional systems.

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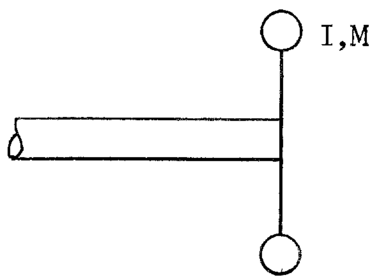
DEVELOPMENT OF COMPUTATIONAL METHOD

The computational method is based on the Holzer-Mykelstad-Prohl (HMP) Method which is discussed in detail in reference 2. The following discussion is purposely brief and intended only to give the reader some idea of what the computer program is doing.

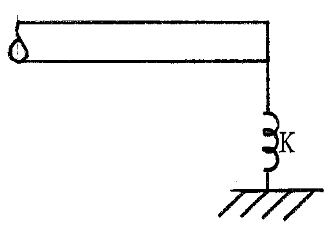
The HMP Method treats any rotor system as being made up of stations. A station is defined as a section of shaft of finite length whose outside and inside diameter remains constant and which has other "parameters" lumped at the right end.

A section of shaft with a large flywheel at the right end would be represented by the following station.



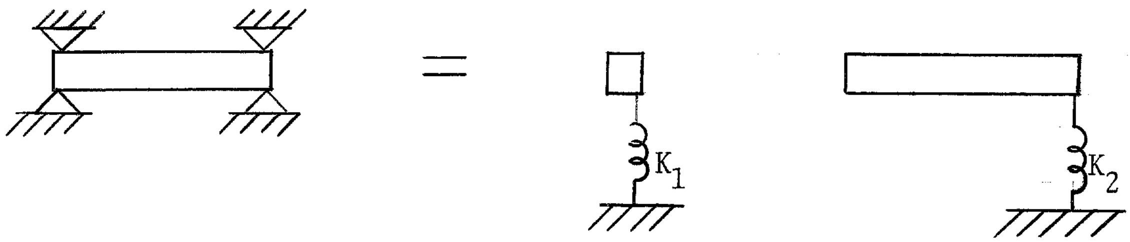
where I = inertia of disk
M = mass of disk

A section of shaft supported in bearings at the right end would be represented by the following station:



where K = restoring force of bearing

A section of shaft supported in bearings at both ends must be represented by two stations:

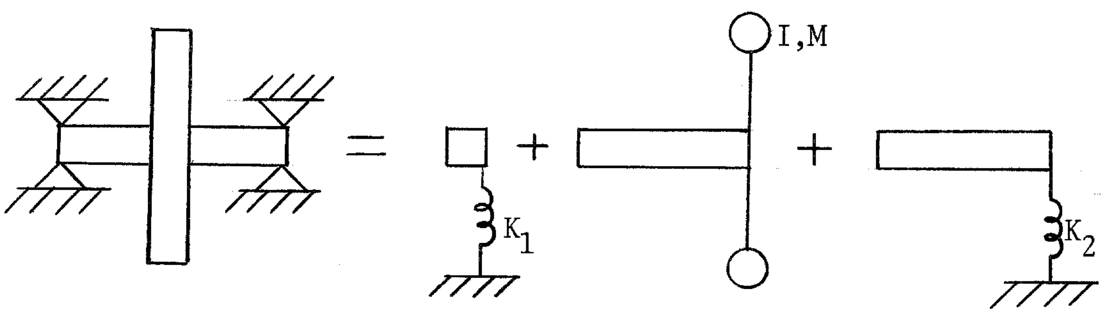


where K_1 = restoring force of left bearing

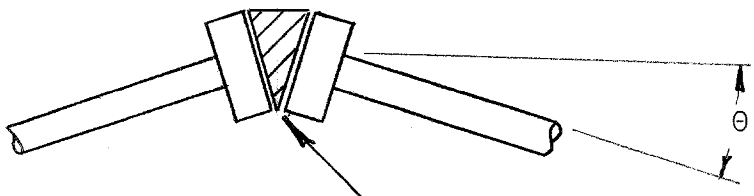
K_2 = restoring force of right bearing

Notice that the left bearing is represented by a station whose length is very small, even though there really is no shaft to the left of the bearing. This is essential for the program to work: a station must have finite length and diameter.

A section of shaft supported in bearings at each end with a flywheel in the middle would be represented by the following stations:



Angular spring forces can also be included. Angular spring forces arise from devices like rubber couplings which resist relative rotations between shaft sections:



The rubber shown in the above coupling will try to straighten out the shaft. Angular restoring forces are usually given by manufacturers in units of pounds per radian.

The HMP Method develops a transfer matrix across a station which relates parameters at the right end of the station to the left end of the station:

$$\begin{bmatrix} U \\ \Theta \\ M \\ V \end{bmatrix}_{\text{Left}} = \begin{bmatrix} & & & \\ & f(\omega^2) & & \\ & & & \end{bmatrix} \begin{bmatrix} U \\ \Theta \\ M \\ V \end{bmatrix}_{\text{Right}}$$

Where:

U = Lateral deflection

Θ = Angular rotation

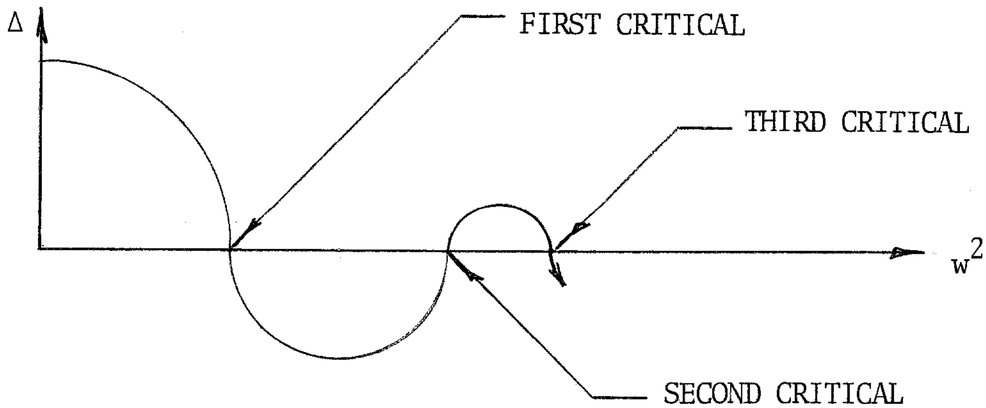
M = Bending moment

V = Shear force

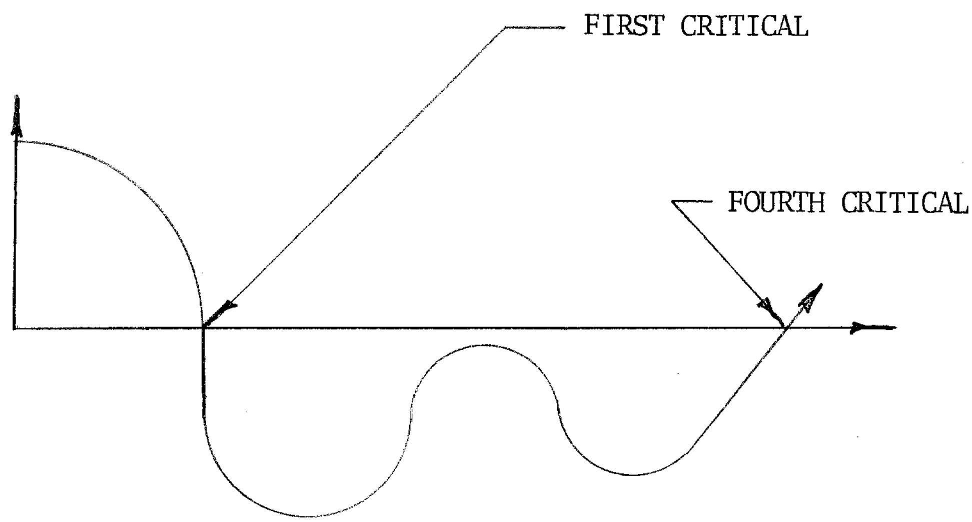
ω = Natural frequency of system

$f(\omega^2)$ = The elements of the matrix are functions of ω^2

Boundary conditions for a free shaft require that the bending moment and shear force be zero at each (free) end of the shaft. By selecting a unit deflection at the left end of shaft and guessing ω^2, the transfer matrix allows one to progress across to the right end of the shaft and see whether the shear force and bending moment really do come out to be zero. If they do the problem is solved (i.e., ω^2 has been found); if they do not, guess a new ω^2 and try again. The program performs the iteration of ω^2. A progressive sequence of guesses for ω^2 will give rise to the following sort of plot:



In the above plot, Δ is the value of a determinant which shall be left undefined for this discussion. Every place the curve crosses the ω^2 axis, a natural frequency occurs. It is appropriate to mention two problems which may occur with this program. The following plot illustrates the problem:



Notice that the second and third criticals were missed entirely simply because small accumulated errors in the program did not allow the curve to cross the ω^2 axis. The program has been written in double precision to help avoid this problem. The other problem occurs when too large an increment is chosen for ω^2; in this case the graph goes above the ω^2 axis and drops down again between two consecutive values for ω^2. When this happens there is no change of sign for the program to detect and the criticals are missed.

One last point concerns gyroscopic effects. A spinning disk (particularly a large one) has an angular momentum vector which resists a torque in any direction. During lateral vibration, if such a torque is exerted on the disk, the disk will resist that torque and so tend to "stiffen" the shaft. This tends to raise the natural frequency of the system. The program will find both the static natural frequency and the dynamic natural frequency which includes gyroscopic effects. Gyroscopic effects are accounted for by substituting -I for I in the input data. It is difficult to see exactly why this works but the following may help. The effective inertia of a disk shows up as $\omega(\omega-2\Omega)I$ where Ω is the precessional frequency. In the static case there is no precession and $\Omega = 0$ leaving $\omega^2 I$ for an effective inertia. In forward synchronous whirl, $\Omega = \omega$ and the effective inertia becomes $\omega^2 I - 2\omega^2 I = -\omega^2 I$. This gives some indication of why substituting a minus I works. Note that it only accounts for forward synchronous whirl.

USE OF PROGRAM WITH EXAMPLE

The input will consist of a description of the shaft and the forcing function. The output will consist of natural frequencies, normal mode shapes and forced vibration response. THE PROGRAM NAME IS LATVIB. Persons interested in using this program should contact the author for a copy of the deck.

LATVIB requires a line printer for output due to the width of the output. To execute the program, the deck should consist of the following cards:

```
JOB NAME.
USER (NUMBER, PASSWORD)
CHARGE CODE.
ATTACH, FTN, FORTRAN, POST PRC/UN = NEW LIBR.
GET (LATVIB = LATVIB)
FTN, I + LATVIB, OPT = 1.
LGO.
7-8-9 CARD
  -
  -      DATA
  -
6-7-8-9 CARD
```

The input has the following format:

CARD	COLUMNS		
1	1-3	Number of problems	(FORMAT I3)
2	1-4	Shaft identification number	(FORMAT I4)
	5-6	4 (this specifies run application)	(FORMAT I2)
3	1-3	Number of shaft sections	(FORMAT I3)
	4-6	Number of frequencies desired	(FORMAT I3)
	7-16	Required accuracy of frequencies in rad/sec	(FORMAT F10.0)
	17-26	Approximation to first natural frequency in rad/sec	(FORMAT F10.0)
	27-36	Frequency increment in rad/sec	(FORMAT F10.0)

CARD	COLUMNS		
		Cards 4, 5, ... , (NS + 3) all have the same format (NS is the number of shaft sections). Each card describes one shaft section. The cards must be arranged in the same order as the shaft sections: Card 4 describes section 1, Card 5 describes section 2, etc.).	
4, 5, ... , (NS + 3)	1-8	Section length in inches	(FORMAT F8.0)
	9-16	Section outer diameter in inches	(FORMAT F8.0)
	17-24	Section inner diameter in inches	(FORMAT F8.0)
	25-32	Section weight density in lb/in^3	(FORMAT F8.0)
	33-40	Section modulus of elasticity in psi	(FORMAT F8.0)
	41-48	Section lumped mass in lb sec^2/in	(FORMAT F8.0)
	49-56	Section lumped diametral inertia in lb sec^2 inches	(FORMAT F8.0)
	57-64	Section linear spring force in lb/in	(FORMAT F8.0)
	65-72	Section angular spring force in in lb/rad	(FORMAT F8.0)

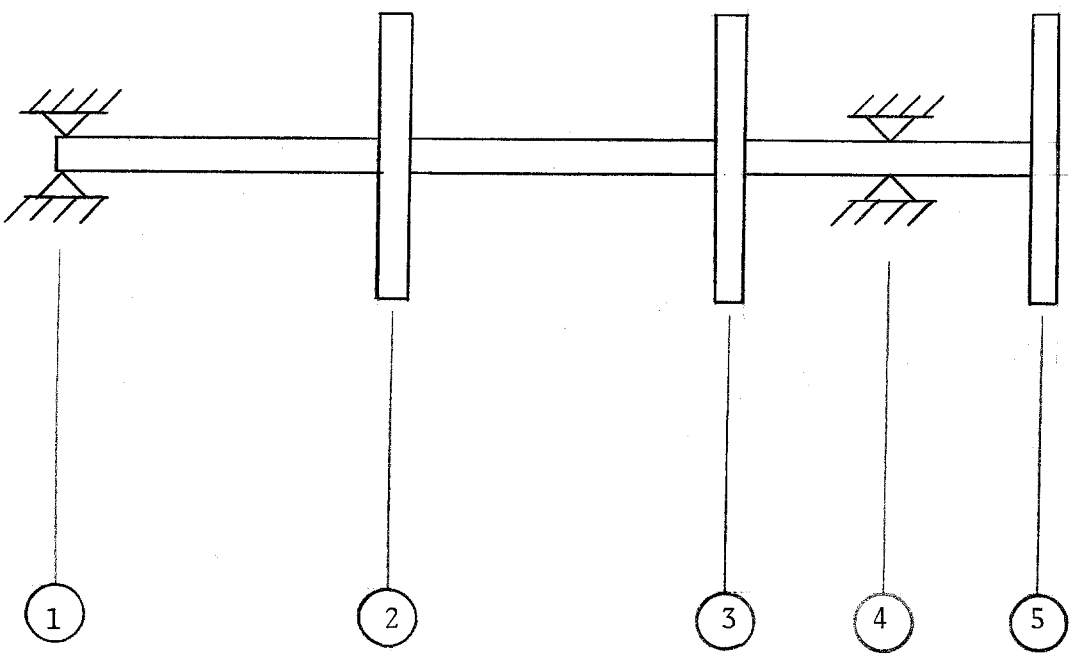
The next cards specify the forcing function at each section. Note that if there is no forcing function at a station, a blank card must be inserted.

NS+4, NS+5, ...,	1-10	Force input at section	(FORMAT F10.0)
2NS+3	11-20	Frequency of force input at section	(FORMAT F10.0)
	21-30	Moment input at section	(FORMAT F10.0)
	31-40	Frequency of moment input at section	(FORMAT F10.0)

The remaining cards specify a quantity called the modal damping factor. This factor is important only in systems where damping has a significant effect on the forced response of the system. In general, the factor will have little effect and values of 0.1-1.0 should be used. There will be one damping factor for each of the frequencies desired (NF). Blank cards must be inserted if no factor is used.

2NS+3, 2NS+4 ... , 2NS+3+NF	1-10	Model damping factor	(FORMAT F10.0)

The following example illustrates the use of the program. The example is a three rotor, two bearing shaft consisting of five stations. The critical speeds will be found for the static case (no gyroscopic effects) and the dynamic case (gyroscopic effects included). The driving force used is 1 lb. at 5000 RPM located at Station 2.



GIVEN

$K_1 = 10^7$ lb/in

$K_4 = 10^7$ lb/in

$M_2 = .02303$ lb s^2/in

$M_3 = .06910$ lb s^2/in

$M_5 = .05757$ lb s^2/in

$J_2 = J_3 = J_5 = .581$ lb s^2 in

The attached copy of the program lists the required input and the corresponding output for the example. Note that in the static case the first natural frequency occurs at 936 radians per second whereas in the dynamic case the first natural frequency occurs at 1025 radians per second. This difference, due to gyroscopic effects, can become quite large for heavy flywheels which are overhung a sizable distance.

REFERENCES

1. Program originally written by Ron Eshleman and E. E. Hahn at IIT Research Institute under contract with the Department of Navy. The author modified the program to run double precision on Fermilab's NOS system.

2. IIT Research Institute, Procedures for Calculating Natural Frequencies of Shafting Systems, Final Report K6086, IIT Research Institute Technology Center, Chicago, Ill., p. 318.

SHAFT IDENTIFICATION NUMBER 2 3 DISK ROTOR INCLUDE GYRO

RUN TYPE 4

THE SHAFT IS COMPOSED OF 5 SECTIONS. 3 NATURAL FREQUENCIES WILL BE COMPUTED WITHIN AN ACCURACY OF PLUS OR MINUS .100000E+01 . IT WAS ESTIMATED THAT THE FIRST NATURAL FREQUENCY WAS GREATER THAN .500000E+03 RADIANS PER SECOND AND THAT NO TWO NATURAL FREQUENCIES WILL BE CLOSER THAN .100000E+01 RADIANS PER SECOND.

LENGTH (IN.)	OUTER DIAMETER (IN.)	INNER DIAMETER (IN.)	WEIGHT DENSITY (LB/IN.3)	MODULUS OF ELASTICITY (LB/IN.2)	LUMPED MASS (LB SEC2/IN.)	LUMPED INERTIA (LB SEC2 IN.)	LINEAR SPRING (LB/IN.)	TORSIONAL SPRING (IN. LB/RADIAN)
.1000E-03	.1000E-03	0.	.2830E+00	.3000E+08	0.	0.	.1000E+08	0.
.1000E+02	.2000E+01	0.	.2830E+00	.3000E+08	.2333E-01	-.5810E+00	0.	0.
.1000E+02	.2000E+01	0.	.2830E+00	.3000E+08	.6910E-01	-.5810E+00	0.	0.
.5000E+01	.2000E+01	0.	.2830E+00	.3000E+08	0.	0.	.1000E+08	0.
.5000E+01	.2000E+01	0.	.2830E+00	.3000E+08	.5757E-01	-.5810E+00	0.	0.

FORCE (LB)	FREQUENCY (RAD/SEC)	MOMENT (IN.LB)	FREQUENCY (RAD/SEC)
0.	0.	0.	0.
.100000E+01	.523600E+03	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.

NATURAL FREQUENCY (RADIANS/SECOND)

MODE SHAPE

SECTION	DISPLACEMENT AT BEGINNING	SLOPE AT BEGINNING	MOMENT AT BEGINNING	SHEAR AT BEGINNING	DISPLACEMENT AT END	SLOPE AT END	MOMENT AT END	SHEAR AT END
.102530E+04								
1	.100000E-02	.297570E-01	0.	0.	.100298E-02	.297570E-01	-.302913E-16	-.606126E-12
2	.100298E-02	.297570E-01	-.302913E-16	.100298E+05	.230132E+00	.970438E-02	.890137E+05	.682269E+04
3	.230132E+00	.970438E-02	.830866E+05	.125047E+04	.152191E+00	-.241604E-01	.674013E+05	-.406844E+04
4	.152191E+00	-.241604E-01	.821577E+05	-.151235E+05	.151054E-02	-.333183E-01	.333200E+04	-.151000E+05
5	.151054E-02	-.333183E-01	.333200E+04	-.294617E+03	-.166055E+00	-.335840E-01	-.153268E-20	-.417500E-02
.344131E+04								
1	.100000E-02	.114732E-01	0.	0.	.100115E-02	.114732E-01	-.341036E-15	-.682203E-11
2	.100115E-02	.114732E-01	-.341036E-15	.100115E+05	.555344E-01	-.468917E-02	.555292E+05	-.157563E+04
3	.555344E-01	-.468917E-02	.977933E+05	-.167238E+05	-.413077E-01	-.340917E-03	-.109554E+06	-.176865E+05
4	-.413077E-01	-.340917E-03	-.107219E+06	.161160E+05	-.153463E-02	.129745E-01	-.150815E+05	.197975E+05
5	-.153463E-02	.129745E-01	-.150815E+05	.445122E+04	.677609E-01	.141823E-01	-.486156E-19	.295176E-03
.901818E+04								
1	.100000E-02	.490786E-02	0.	0.	.100049E-02	.490786E-02	-.234151E-14	-.468340E-10
2	.100049E-02	.490786E-02	-.234151E-14	.100049E+05	.938429E-02	-.260395E-02	-.112402E+05	-.146963E+05
3	.938429E-02	-.260395E-02	.111800E+06	-.322751E+05	-.421375E-01	.612342E-02	-.610064E+05	.183362E+05
4	-.421375E-01	.612342E-02	-.350346E+06	.255134E+06	-.574901E-01	-.594193E-01	.981464E+06	.276304E+06
5	-.574901E-01	-.594193E-01	.981464E+06	-.298597E+05	-.641133E+00	-.137642E+00	.708823E-18	-.357044E-02

FORCED VIBRATIONS

SECTION	DISPLACEMENT AT BEGINNING	MOMENT AT BEGINNING	SHEAR AT BEGINNING	DISPLACEMENT AT END	MOMENT AT END	SHEAR AT END
1	.632715E-07	0.	0.	.684539E-07	-.560402E-20	-.112112E-15
2	.684539E-07	-.560402E-20	.684539E+00	.137334E-04	.559728E+01	.370561E+00
3	.137334E-04	.572595E+01	-.118349E+00	.819853E-05	.259749E+01	-.431719E+00
4	.819853E-05	.346200E+01	-.676719E+00	.681815E-07	.269701E-01	-.690485E+00
5	.681815E-07	.269701E-01	-.867003E-02	-.869411E-05	-.531831E-24	-.234431E-06

ANALYSIS OF COMPLEX SHAFTING SYSTEMS

SHAFT IDENTIFICATION NUMBER 1 3 DISK ROTOR NEGLECT GYRO

RUN TYPE 4

THE SHAFT IS COMPOSED OF 5 SECTIONS. 3 NATURAL FREQUENCIES WILL BE COMPUTED WITHIN AN ACCURACY OF PLUS OR MINUS .100000E+01 . IT WAS ESTIMATED THAT THE FIRST NATURAL FREQUENCY WAS GREATER THAN .500000E+03 RADIANS PER SECOND AND THAT NO TWO NATURAL FREQUENCIES WILL BE CLOSER THAN .100000E+01 RADIANS PER SECOND.

LENGTH (IN.)	OUTER DIAMETER (IN.)	INNER DIAMETER (IN.)	WEIGHT DENSITY (LB/IN.3)	MODULUS OF ELASTICITY (LB/IN.2)	LUMPED MASS (LB SEC2/IN.)	LUMPED INERTIA (LB SEC2 IN.)	LINEAR SPRING (LB/IN.)	TORSIONAL SPRING (IN. LB/RADIAN)
.1000E-03	.1000E-03	0.	.2830E+00	.3000E+08	0.	0.	.1000E+08	0.
.1000E+02	.2000E+01	0.	.2830E+00	.3000E+08	.2333E-01	.5810E+00	0.	0.
.1000E+02	.2000E+01	0.	.2830E+00	.3000E+08	.5910E-01	.5810E+00	0.	0.
.5000E+01	.2000E+01	0.	.2830E+00	.3000E+08	0.	0.	.1000E+08	0.
.5000E+01	.2000E+01	0.	.2830E+00	.3000E+08	.5757E-01	.5810E+00	0.	0.

FORCE (LB)	FREQUENCY (RAD/SEC)	MOMENT (IN.LB)	FREQUENCY (RAD/SEC)
0.	0.	0.	0.
.100000E+01	.52360E+03	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.

NATURAL FREQUENCY (RADIANS/SECOND)

MODE SHAPE

SECTION	DISPLACEMENT AT BEGINNING	SLOPE AT BEGINNING	MOMENT AT BEGINNING	SHEAR AT BEGINNING	DISPLACEMENT AT END	SLOPE AT END	MOMENT AT END	SHEAR AT END
.936579E+03								
1	.100000E-02	.324711E-01	0.	0.	.100325E-02	.324711E-01	-.252782E-16	-.505837E-12
2	.100325E-02	.324711E-01	-.252782E-16	.100325E+05	.257034E+00	.123058E-01	.899266E+05	.768312E+04
3	.257034E+00	.123058E-01	.962682E+05	.189000E+04	.172001E+00	-.287476E-01	.885735E+05	-.314071E+04
4	.172001E+00	-.287476E-01	.739225E+05	-.135661E+05	.135521E-02	-.370003E-01	.309313E+04	-.144763E+05
5	.135521E-02	-.370003E-01	.309313E+04	-.924198E+03	-.184550E+00	-.372468E-01	-.706914E-20	-.267735E-02
.312099E+04								
1	.100000E-02	.103371E-01	0.	0.	.100103E-02	.103371E-01	-.280492E-15	-.561080E-11
2	.100103E-02	.103371E-01	-.280492E-15	.100103E+05	.413860E-01	-.718394E-02	.678329E+05	.185025E+04
3	.413860E-01	-.718394E-02	.271771E+05	-.742487E+04	-.257442E-01	.312709E-03	-.626239E+05	-.777724E+04
4	-.257442E-01	.312709E-03	-.608542E+05	.955018E+04	-.923986E-03	.771231E-02	-.729157E+04	.113931E+05
5	-.923985E-03	.771231E-02	-.729157E+04	.215324E+04	.397754E-01	.829602E-02	.134193E-20	.100824E-02
.501945E+04								
1	.100000E-02	.518858E-02	0.	0.	.100052E-02	.518858E-02	-.725394E-15	-.145091E-10
2	.100052E-02	.518858E-02	-.725394E-15	.100052E+05	-.819122E-02	-.116779E-01	.659985E+05	.363402E+04
3	-.819122E-02	-.116779E-01	-.104945E+06	.938749E+04	.192765E-01	.743575E-02	.286679E+05	.147250E+05
4	.192765E-01	.743575E-02	.137514E+06	-.188343E+05	.158787E-02	-.106174E-01	.279157E+05	-.240555E+05
5	.158787E-02	-.106174E-01	.279157E+05	-.817680E+04	-.597086E-01	-.128608E-01	.239519E-19	-.287588E-02

FORCED VIBRATIONS

SECTION	DISPLACEMENT AT BEGINNING	MOMENT AT BEGINNING	SHEAR AT BEGINNING	DISPLACEMENT AT END	MOMENT AT END	SHEAR AT END
1	.599644E-07	0.	0.	.601290E-07	-.471632E-20	-.943522E-16
2	.601290E-07	-.471632E-20	.601290E+00	.124791E-04	.509278E+01	.351591E+00
3	.124791E-04	.491443E+01	-.260105E-01	.756012E-05	.316571E+01	-.267974E+00
4	.756012E-05	.243795E+01	-.475311E+00	.481235E-07	.177742E-01	-.487029E+00
5	.481235E-07	.177742E-01	-.579384E-02	-.790845E-05	-.324003E-24	-.107080E-06